Parallel connection of capacitors is widely used in power electronics to decrease high frequency ripples and current stress, to decrease power dissipation and operating temperature, to shape frequency response, and to boost reliability. **Alexander Asinovski, Principal Engineer, Murata Power Solutions, Mansfield, USA**

Parallel connection of capacitors is widely used in power electronics to decrease high frequency ripples and current stress, to decrease power dissipation and operating temperature, to shape frequency response, and to boost reliability. Main questions a designer faces with regard to the parallel connection of capacitors are: What are equivalent capacitance and ESR (electric series resistance) values? What is high frequency ripple voltage? What are individual RMS currents?

If all capacitors in the parallel connection are identical with equal capacitance values, \( C = C_k \), \( k = 1,2,\ldots,N \) and equal ESR values \( R = R_k \), the answers are obvious: \( C \) is directly proportional to the number of capacitors \( N \), \( C = NC_k \), \( k = 1,2,\ldots,N \). If all capacitors in the parallel connection are not identical, with different capacitance and ESR values, \( C_k \) and \( R_k \), the answers are not obvious: \( C \) is inversely proportional to \( N \), \( C = C_k / N \), ripple voltage \( V \) (RMS value) for a sinusoidal current excitation \( i(t) = \sqrt{2} \sin(2\pi ft) \) with frequency \( f \), and RMS value \( I \) is

\[
V = I \sqrt{R_{se}^2 + X_{sk}^2},
\]

where \( X_{sk} = 1/(2\pi f C_k) \) is the reactance of the equivalent capacitor \( C_k \) and individual RMS currents in the capacitors are identical: \( I_k = I / N \).

In case capacitors in the parallel connection are not identical, with different capacitance \( C_k \) and ESR \( R_k \) values, the solution to the problem is not trivial. The direct approach would be obtaining an analytical expression for the input impedance of the parallel connection in the algebraic form \( Z = R + j \cdot \Im Z \) and using the formulas \( R_{pk} = R_{k} \), \( X_{sk} = \Im Z_k \) and \( C_{pk} = C_{k} / N \). A less complicated approach taken below is based on the conversion of series, \( C_{pk} \), \( R_{pk} \) connections to equivalent parallel \( C_k \), \( R_k \) connections. To obtain relationships between \( R_{pk} \) and \( R_k \), and also between \( C_{pk} \) and \( C_k \), set admittance \( Y_k \) of the parallel \( C_k \), \( R_k \) and admittance \( Y_{se} \) of the series \( C_{se} \), \( R_{se} \) connections equal to each other: \( Y_k = Y_{se} \), \( \Re(Y_k) = \Re(Y_{se}) \) and \( \Im(Y_k) = \Im(Y_{se}) \). It follows:

\[
C_{pk} = C_se \left[ 1 + \left( R_{pk} / X_{sk} \right)^2 \right], \quad (2)
\]

\[
R_{pk} = \left( R_{se}^2 + X_{sk}^2 \right) / R_{sk}, \quad (3)
\]

where \( X_{sk} = 1/(2\pi f C_k) \) is the reactance of the individual capacitor.

After individual parallel capacitance \( C_k \) and resistance \( R_k \) values are calculated according to (2) and (3), equivalent parallel capacitance \( C_{pe} \) can be easily found as the sum of \( C_k \):

\[
C_{pe} = \sum_{k=1}^{N} C_{pk} \quad (5)
\]

and real part of equivalent admittance can be found as the sum of admittances 1/\( R_{pk} \). \( R_{pk} \) can be obtained as a reverse value of that sum:

\[
R_{pk} = 1 / \left( \sum_{k=1}^{N} \left[ 1 / R_{pk} \right] \right), \quad (6)
\]

Equivalent series capacitance \( C_{se} \) and ESR \( R_{se} \) of the system can be found by conversion of the parallel \( C_{pe} \), \( R_{pe} \) connection to the equivalent series connection \( C_{se} \), \( R_{se} \). To obtain relationships between \( C_k \) and \( C_{se} \), and also between \( R_k \) and \( R_{se} \), set impedance \( Z_{se} \) of the parallel \( C_{se} \), \( R_{se} \) and impedance \( Z_{sk} \) of the series \( C_{sk} \), \( R_{sk} \) connections equal to each other: \( Z_{se} = Z_{sk} \), \( \Re(Z_{se}) = \Re(Z_{sk}) \) and \( \Im(Z_{se}) = \Im(Z_{sk}) \). It follows:

\[
C_{se} = C_{pk} \left[ 1 + \left( R_{pk} / X_{sk} \right)^2 \right], \quad (7)
\]

\[
R_{se} = R_{pk} \left[ 1 + \left( R_{pk} / X_{sk} \right)^2 \right], \quad (8)
\]

where

\[
X_{sk} = 1/(2\pi f C_k),
\]

is reactance of the equivalent parallel capacitor \( C_k \).

Based on the analysis presented above, calculation procedure for equivalent series capacitance, ESR, voltage ripples, and RMS currents in the capacitors is as follows:

1. Calculate reactivities of individual capacitances according to formula (4).
2. Determine equivalent parallel parameters \( C_{pe} \), \( R_{pe} \) of the capacitors based on equations (2) and (5).
3. Calculate equivalent parallel capacitance \( C_{se} \) of the structure, its reactance \( X_{se} \), and equivalent parallel resistance \( R_{se} \) according to formulas (5), (7), and (8).
4. Calculate equivalent series capacitance \( C_{sk} \) and ESR \( R_{sk} \) of the structure according to formulas (7) and (8).
5. Obtain RMS ripple voltage \( V \) using equation (1).
6. Calculate RMS currents \( I_k \) in the capacitors based on the formula

\[
I_k = V / \sqrt{R_{sk}^2 + X_{sk}^2}.
\]

(10)

It is worthwhile to note that ESR values are strong functions of frequency. A designer should use ESR data supplied by capacitor manufacturers at a given frequency of operation. An example of a comprehensive source of data for ceramic and polymer aluminum electrolytic capacitors is found on the Murata Manufacturing Co., Ltd. (MMC) website http://ds.murata.co.jp/software/simsurfing/en-us/index.html.

To illustrate the calculation procedure let’s determine equivalent parameters, voltage ripple, and current distribution for a parallel connection of three ceramic capacitors GRM218R601226ME39L and one polymer capacitor ESASD40U107M015K00 from MMC.

Assuming the following input data:

\( f = 200 \text{ kHz} \), \( C_s = C = 22 \mu F \), \( R_s = R = 4 \text{ m\Omega} \), \( C_{se} = 100 \mu F \), \( C_{sk} = 8 \text{ m\Omega} \), \( I = 2A \).
1. For reactance of each individual capacitance according to formula (4) we have:
   \[ X_{s1} = X_{s2} = X_{s3} = 3.6 \, \text{m} \Omega, \quad X_{s4} = 0.8 \, \text{m} \Omega. \]

2. Equivalent parallel parameters \( C_{pl}, R_{pl} \) of the capacitors based on formulas (2) and (3) are:
   \[ C_{p1} = C_{p2} = C_{p3} = 21.7 \, \mu \text{F}, \quad R_{p1} = R_{p2} = R_{p3} = 331 \, \text{m} \Omega, \quad C_{p4} = 49.7 \, \mu \text{F}, \quad R_{p4} = 16 \, \text{m} \Omega. \]

3. For equivalent parallel capacitance \( C_{pe} \), its reactance \( X_{pe} \), and equivalent parallel resistance \( R_{pe} \) of the structure according to formulas (5), (9), and (6) we calculate:
   \[ C_{pe} = 115 \, \mu \text{F}, \quad X_{pe} = 6.9 \, \text{m} \Omega, \quad R_{pe} = 13.9 \, \text{m} \Omega. \]

4. Equivalent series capacitance \( C_s \) and ESR \( R_s \) according to formulas (7) and (8) are:
   \[ C_s = 143.4 \, \mu \text{F}, \quad R_s = 2.76 \, \text{m} \Omega. \]

5. For RMS ripple voltage based on equation (1) we obtain: \( V = 12.4 \, \text{mV}. \)

6. RMS currents according to formula (10) in ceramic and polymer capacitors are respectively:
   \[ I_1 = I_2 = I_3 = 341 \, \text{mA}, \quad I_4 = 1.1 \, \text{A}. \]

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